Matching definite integrals with limits of Riemann sums

<table>
<tr><th>Given</th><th>Requested</th><th>Steps</th></tr>
<tr><td>$\int_a^b f(x)\,\mathrm{d}x$</td><td>$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x$</td><td>1. Fill out top of the rectangular approximation method worksheet for n subintervals (pause after filling in 3 rows in the table).
2. In the fourth row, write ellipses ("…") for all entries.
3. In the fifth row, write out expressions in all table cells for subinterval i.
4. Copy Δx from near the top of the table and $f(x_i^*)$ from the 5th row of the table into
$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x$$</td></tr>
<tr><td>$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x$</td><td>$\int_a^b f(x)\,\mathrm{d}x$</td><td>1. Speculate that the definite integral could run from $a = 0$ to $b = 1$.
2. Speculate that the given limit is a limit of a RRAM with n subintervals.
3. Conclude that Δx would have to equal $\frac{1}{n}$.
4. Try to find one copy of the factor $\frac{1}{n}$ inside the summation that could play the role of Δx.
   a. Circle the factor $\frac{1}{n}$.
   b. Label the quantity $\frac{1}{n}$ as Δx.
5. Try to find a copy or copies of the quantity $x_i^* = \frac{i}{n}$ inside the remaining factor in the summation.
   a. Circle the quantity $\frac{i}{n}$.
   b. Label the quantity $\frac{i}{n}$ as x_i^*.
6. Label the contents of the summation (other than the single factor of Δx already circled in step 4) as $f(x_i^*)$.
7. Write out an expression for $f(x_i^*)$ in terms of x_i^*.
8. Use the $f(x_i^*)$ you just obtained to fill in
$$\int_0^1 f(x)\,\mathrm{d}x$$
9. If you are doing a multiple choice question, check whether the definite integral exactly matches one of the choices.
10. If your definite integral does not exactly match one of the choices, think about whether transformations/identities for definite integrals can be used to show that the definite integral you obtained is equivalent to one of the definite integrals in the choices.</td></tr>
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